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Number Representation and Calculation: An Overview of Chapter 4 of Thinking Mathematically

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NUMBER REPRESENTATION AND CALCULATION

An overview of Chapter 4 of *Thinking Mathematically*. (Math 107)

By Emily Pawlicki

Basic Terms:

A **number** answers the question "How many?"

Numbers are represented by **numerals**. (ie, 9, IX, 23)

A **numeration system** is a set of numerals with rules on how to combine them to form actual numbers.

Exponential Notation:

If you have a base, *b*, multiplied by itself *n* times, you can write this as b^n. This is called an **exponential expression.**

Exponential notation is used to shorten writing numbers.

EXAMPLES: $10 \times 10 \times 10 = 10^3$ $5 \times 5 = 5^2$ $6 = 6^1$

EARLY NUMERATION SYSTEMS:

There were many early numeration systems. All had strengths and weaknesses. We will look at two precursors to our own system:

The Egyptian Numeration System

and

The Mayan Numeration System

The Egyptian Numeration System:

The oldest Egyptian numeration system arose around 3400 B.C.

The Egyptians used hieroglyphs as numerals.

The numerals were in powers of ten, like our system.

It’s inefficient because you have to write out each numeral the required number of times.

It’s an **additive system**, meaning a number is the sum of the values of its numerals.

Egyptian Numeral Descriptions:

The symbol for 1 is a staff

The symbol for 10 is a heel bone

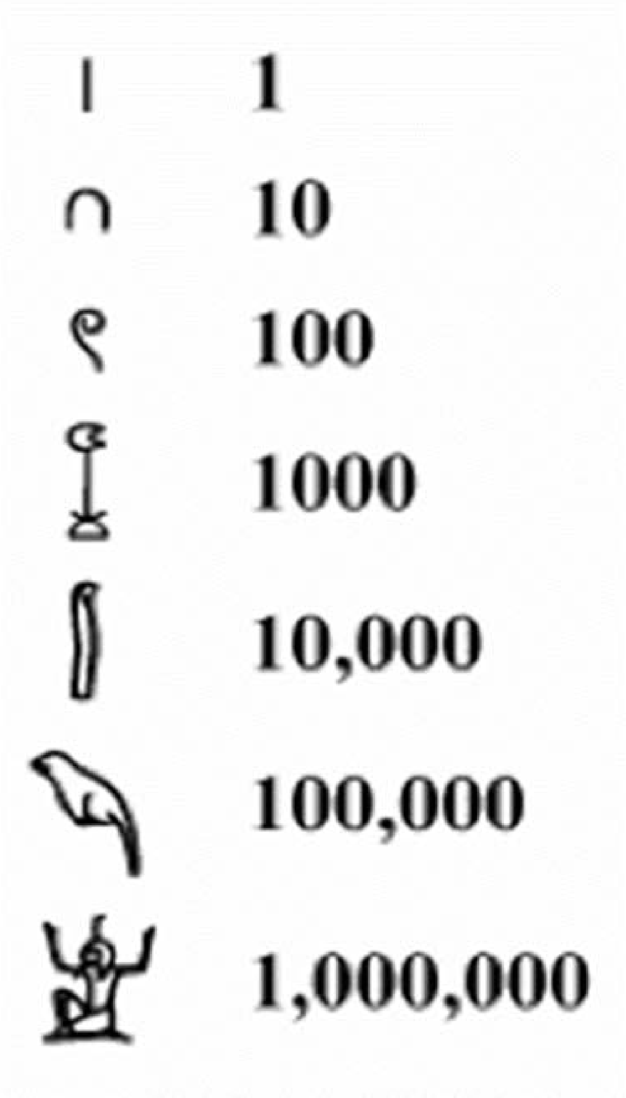
The symbol for 100 is a spiral

The symbol for 1000 is a lotus blossom

The symbol for 10,000 is a pointing finger

The symbol for 100,000 is a tadpole

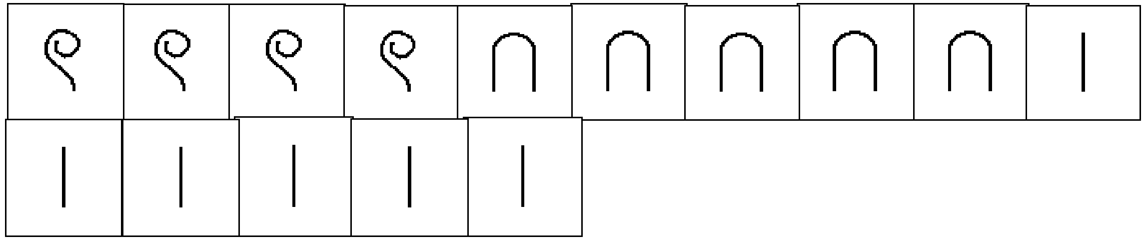
The symbol for 1,000,000 is an astonished person

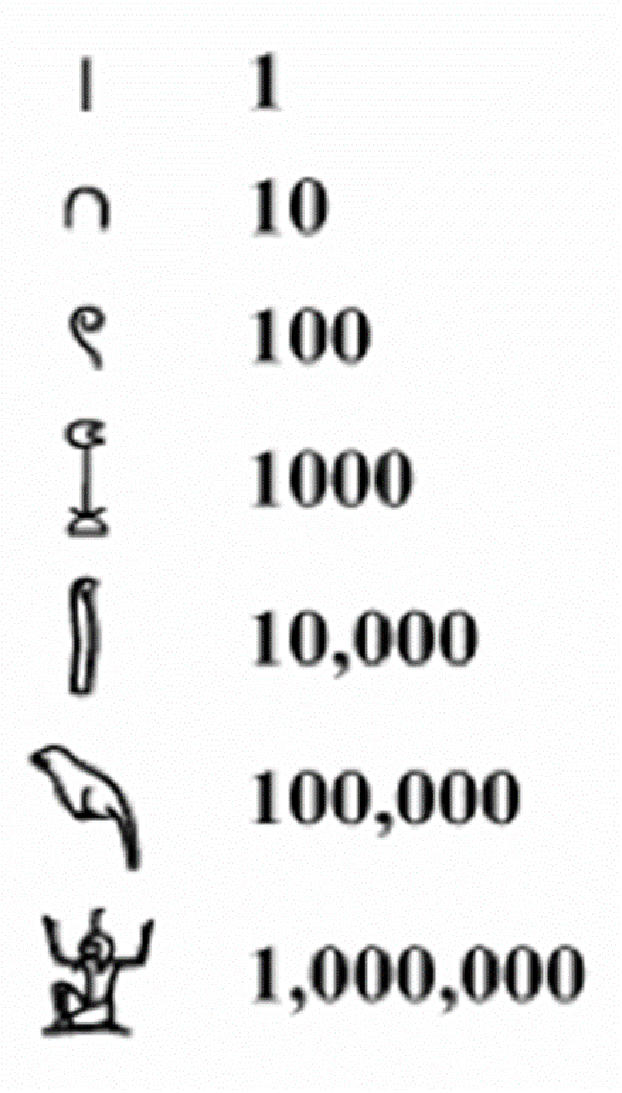


Writing in Egyptian Numerals:

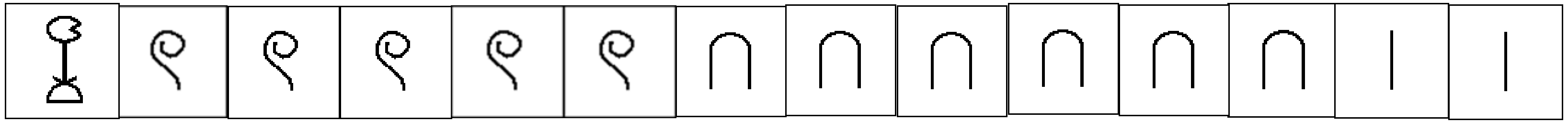
EXAMPLE: Writing 456 in Egyptian numerals

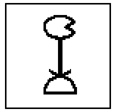
would look like:

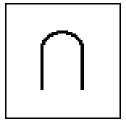




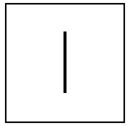
Converting from Egyptian Numerals:



 Is the symbol for 1000.

 Is the symbol for 100

 Is the symbol for 10

 Is the symbol for 1

There is one thousand symbol, five hundred symbols, six ten symbols, and two one symbols.

1000+1oo+100+100+100+100+10+10+10+10+10+10+1+1= **1,562.**

The Mayan Numeration System:

The Maya lived in Central America between 300 and 1000 A.D.

The symbols used as numerals are combinations of dots and lines.

Their system is the first one known to have a symbol for zero: a "coin" or "shell" shape.

The numerals are expressed vertically with the ones place at the bottom.

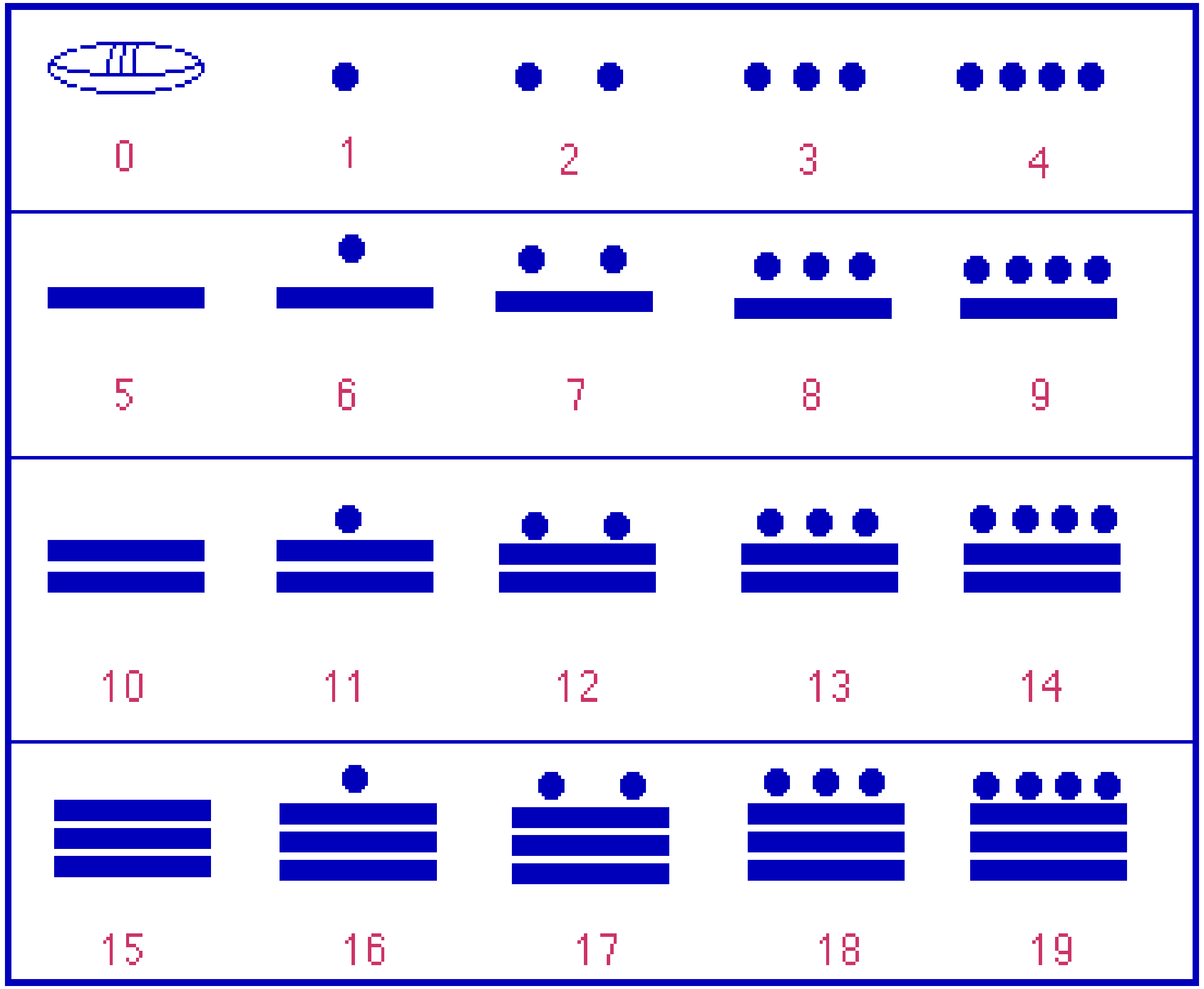
The Mayan Base System:

The Maya used a unique system for place values: 1, 20, 18×20, 18×20^2…

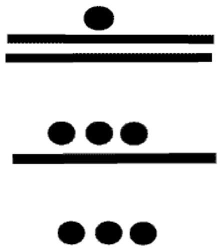
It's guessed the Maya used base 20 because they counted on their fingers and toes.

It is also guessed the system added the 18 to make the number of days in their calendar year (360) a natural part of the system.

The Mayan Numerals:



Example:



is equal to 4123.

 is 11,  is 8 and  is 3.

 is in the first row, making it: $11^{x}18^{x}20= 3960$

 is in the second row, making it: $8^{x}20= 160$

 is in the third row, making it: $3^{x}1= 3$

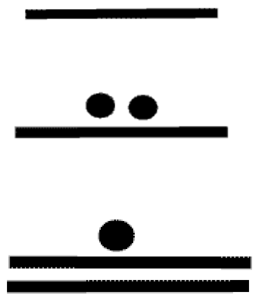
3960+160+3= 4123

Converting to Mayan Numerals:

Converting 1951 to Mayan Numerals:

Try to figure out the largest place value to go in. It's $18^{x}20$ in this case. It goes in 5 times with a remainder of 151. Divide that by the next place value, 20, to get 7, with a final remainder of 11.

The final answer is expressed like:



Our Numerals:

We use numerals that were invented in India and came to Europe with the Arabs. They're called **Hindu-Arabic numerals**, and are made of ten symbols, called **digits** (Latin for *fingers*): 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

The Hindu-Arabic system is a place-value system based on powers of ten, like the Egyptians' system. It's guessed this is because of people counting on fingers instead of fingers and toes like the Maya. We call our system Base Ten.

Working in Base Ten:

Base ten is written in powers of ten. The place farthest to the right is the ones place, the next place is the tens (or 10^1), the next is 10^2, the next 10^3, the next 10^4, and so on.

These powers can be written as: 1, 10, $10^{x}10$, $10^{x}10^{x}10$, and $10^{x}10^{x}10^{x}10$, and so on.

This can also be written by the multiplied out values: 1, 10, 100, 1000, 10,000, and so on.

Example: Ways of Writing Base Ten:

Writing 12,345 in base ten.

WAY 1: 1(10,000) + 2(1000) + 3(100) + 4(10) + 5(1)

WAY 2: 1(10$\times$10$\times$10$\times$10) + 2(10$\times$10$\times$10) + 3(10$\times$10) + 4(10) + 5(1)

WAY 3: 1(10^4) + 2(10^3) + 3(10^2) + 4(10^1) + 5(1)

Bases:

A **base** is the number of single numerals able to be used in a system *and* the number used to designate place values.

Other bases include: base two (or **binary**, used in computers by using 1 as “on” and 0 as “off.”), base four, base eight, and base six.

In all bases, digit symbols go to one less than the base. (Example: base six digits are: 0, 1, 2, 3, 4, and 5,)

Converting to Base Ten:

Look at the place values of the number you want to convert. (i.e. the ones, tens, hundreds places in base ten.)

Multiply each number by the place value it is in. (i.e. if there is a four in the hundreds place, you will get 400.)

Sum these numbers.

EXAMPLE: Conversion to Base Ten

4543_8 to base ten

PLACE VALUES: $4 \rightarrow 8^3$ $5 \rightarrow 8^2$ $4 \rightarrow 8^1$ $3 \rightarrow 1$

MULTIPLY: $4 \times 8^3 = 2048$ $5 \times 8^2 = 320$ $4 \times 8^1 = 32$ $3 \times 1 = 3$

ADD: $2048 + 320 + 32 + 3 = 2403$

ANSWER: 4543_8 is 2403_{10}

Converting from Base Ten:

Find the place values in the base you are converting to.

Take the highest place value less than the number in base ten and divide that place value out. Divide the remainder by the next place value. Continue until you get to the ones place.

Take the numbers to create the alternate base’s corresponding number.

EXAMPLE: Converting from Base Ten

234_{10} to base four

PLACE VALUES: 4^4, 4^3, 4^2, 4^1, and 1. (OR: 256, 64, 16, 4, 1)

Highest place value to divide into 234 is 64. (**3** times.)

Remainder: 42   16 goes into 42 **2** times with a remainder of 10

4 goes into 10 **2** times with a remainder of **2**.

ANSWER: 234_{10} is 3222_4.

Addition in Alternative Bases:

Add the numbers in the right-most column.

If you get a digit symbol not present in the base, divide your answer by the base. (i.e. 8 base 6, divides to be one 6 and two remaining.)

Keep the remainder as your answer and add the base(s) to your next column.

Continue adding until you have done all columns.

EXAMPLE: Addition in Alternate Bases.

$111_2 + 101_2$

Add the 1 and 1 to get a 2. There is no “2” digit, so it carries to the next place, leaving a 0 for that column. Add the 1 and 0 and the carried 1 to get 2 again. Carry that one 2 to the next column, leaving a 0. Add the 1 and 1 and the carried 1 to get 3. Divide out 2 to get a remainder of 1. Leave the 1 set of two and carry the 1 over to the next column to get a 1 there.

ANSWER: 1100_2

Subtraction in Alternate Bases:

Start by subtracting the right-most column.

If the top number is smaller than the bottom number, you will need to borrow from the next column over. Borrow the number of the base. (i.e. in base four you borrow 4)

Continue through the remaining columns.

EXAMPLE: Subtraction in Alternate Bases

$232_6 - 143_6$

Subtract the right-most column. You can't subtract 3 from 2, so take a 6 from the 3 in the 6 place, leaving a 2. Add the 6 to 2 to get 8. Subtract 3 from 8 to get 5. In the next column, you can't subtract 4 from 2, so borrow a 6 from the 2 to leave 1 in the 6^2 place. Add the 6 to the 2 to get 8. Subtract 4 from the 8 to get 4. In the next column, you have two ones that subtract to 0.

ANSWER: 45_6

Multiplication in Alternate Bases:

Multiply as you would in base ten.

However, when you get an answer, divide out the base of the system you are working in. (i.e. if you get a 6 in base 4 you get one group of four with two left over for a value of 12base four)

Leave any remainder, and carry the full base groups to the next column.

Multiply the same way, just adding in the carried number.

Continue through all columns.

EXAMPLE: Multiplying Alternate Bases

$43_5 \times 2_5$

Multiply the 2 times the 3 to get 6. Divide out the group of 5 and carry it to the next column, leaving the one as a remainder.

Multiple the 2 times the 4 to get 8. Divide out the group of 5 and carry it over to the next column, leaving the 4 as a remainder.

ANSWER: 141_5

Division in Alternate Bases:

Divide just as you would in base ten. However, what you divide by are the multiplications of the numbers in the base you are working in. You take the largest number that is divisible into the number, divide it out, and then divide into the remainder. Continue until you have 0 left.

EXAMPLE: Division of Alternate Bases

$100_4 \div 2_4$

Make a table of multiples of **2_4. : 2_4 ^{x}1, $2_4{}^x$2, $2_4{}^x$3…** Remember: in base four, you can only use numerals 0-3. So, 2_4 ^{x}1=2, but $2_4{}^x$2=10 because 2^x2 is four, which isn't a numeral in base four, so is expressed as one group of four and zero ones.

Begin by dividing 10 by 2. The nearest multiple is $2_4{}^x$2=10, so divide out the 2 and place it up top and subtract the 10 from 10 to get 0. Bring down the 0 from the 100 to get 00. 2 doesn't go in to 0, so place a zero up by the 2.

ANSWER: 20_4

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