A with Honors Projects Honors Program

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Brewster's Angle

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Honors Project Report

PHY-143-Honors Project

Brewster's Angle

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1. Introduction

Introduction: The sun and lasers send out unpolarized light, which is light mixed with all directions of polarization. An interesting phenomenon occurs when this unpolarized light hits a smooth surface. If the incident angle is just right, the reflected light is completely polarized. This special incident angle is called "Brewster's angle".

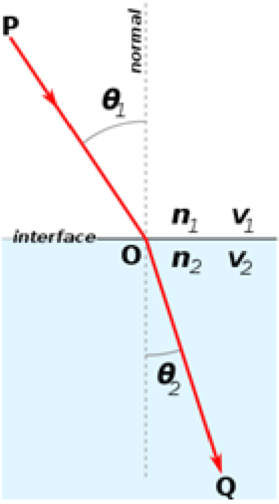
In this project, I experimentally measure the Brewster's angle of various materials, with different values of index of refraction. I measure that of popular materials for this experiment, such as water and glass, but also that of less popular materials, such as orange juice and alcohol.

2. Background Theory

1) Refraction and Snell's Law:
   Refraction of light is the bending of light when light travels from one material to another, where the direction of travel changes at the interface.

   The reason behind this is the speed difference of light. Light has speed, and this speed depends on the medium. This is analogous to when a car runs between paved and unpaved roads -- the speed of each wheel varies depending on the surface of the road, which leads to a change in direction of the car.

   In the case of light, when a beam of light travels from medium 1 (index of refraction n_1, speed v_1, wavelength λ_1) to medium 2 (index of refraction n_2, speed v_2, wavelength λ_2) with incident angle θ_1 and refracted angle θ_2, the relationship is described by Snell's Law.

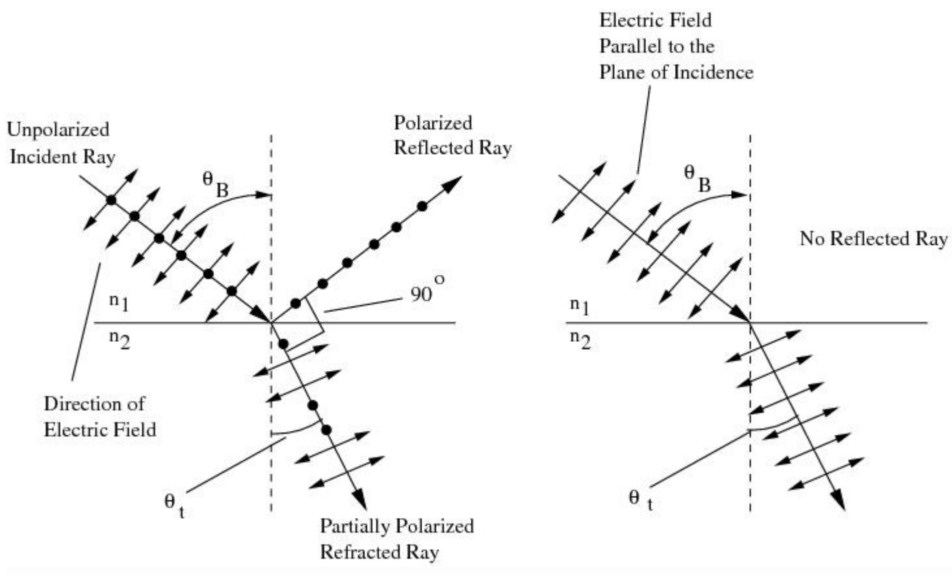


[1]

$$\frac{sin\theta_2}{sin\theta_1} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2} \quad (1)$$

This bending of light changes the appearance of depth of water when seen from above. Even when objects in water look very close, they are deeper than we think. Because of that, in rivers with clean water, people sometimes go in and drown.

2) Brewster's Angle: When a light beam is incident on a surface of another medium, light that is polarized parallel to the surface gets reflected more than light that is polarized in the other direction. In fact, when the incident angle is just right, the reflected light gets completely polarized. This particular angle of incidence is called Brewster's angle. At this angle, reflected and transmitted rays form is 90° as in the following figure.



[2]

By using Snell's Law, $n_1 sin\theta_1 = n_2 sin\theta_2$, if the incident angle in Brewster's angle $\theta_1 = \theta_B$, then the angle of refraction must be 90° - θ_B, as one can see from the figure above. Hence, we find: $n_1 sin\theta_B = n_2 \sin(90° - \theta_B) = n_2 cos\theta_b$. Therefore, we can get the Brewster's angle in terms of $n_1$and n_2:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right). \qquad (2)$$

3. Equipment and Experimental Procedure

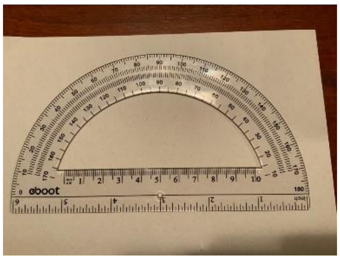
To confirm the aforementioned theories, I prepared three experiments. The first and second experiments are form confirming Snell Law. I used angles of incidence and refraction for experiment 1, and I used critical angle for experiment 2. By comparing the results in experiment 1 and the experiment 2, I confirm the concept of Snell's Law.

In the third experiment, I experimentally measured Brewster's angle of various materials. I used the fact that reflected light is completely polarized when the incident angle is Brewster's angle; if a light beam is polarized, the beam can be blocked with a polarizer. I searched for the incident angle where reflected light gets blocked by a polarizer.

Equipment: Laser pointer, polarizing lens, protractor, various materials for second medium


Polarizing lens


Laser pointer


Protractor

Experimental Procedure:

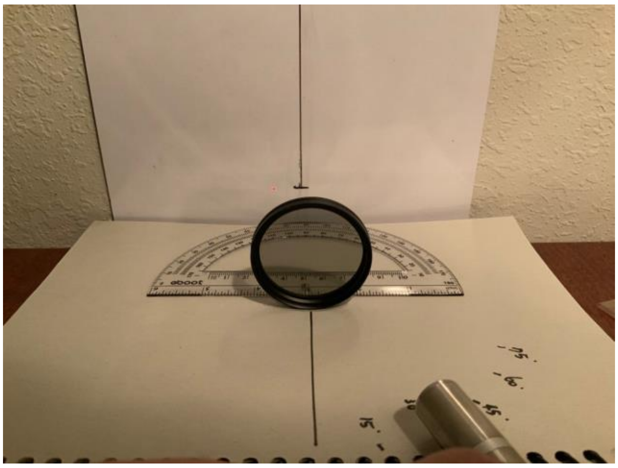
1) Experiment 1: Confirming Snell's law through the relationship between angles of incidence and refraction.

   A. Place the angled disc (Protractor) on the polarizing lens vertically and use the laser pointer to illuminate the light.

   B. Measure the angle of refraction when the angle of incidence is 15 degrees, 30 degrees, 45 degrees, 60 degrees, and 75 degrees.

   C. Adjust the angle of incidence until the refraction light is not visible as I continuously change the angle of incidence.

   D. Measure the angle of refraction at the threshold.



2) Experiment 2: Measure the index of refraction of various materials.

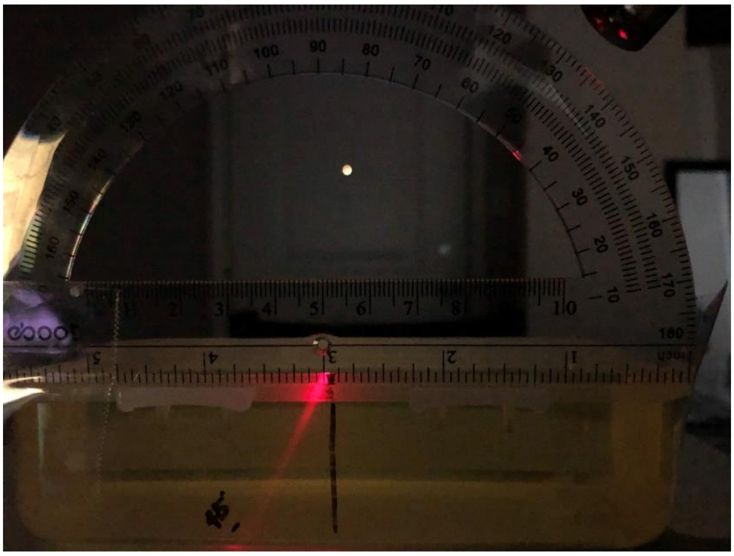
   A. Set up materials, such as water, juice (mixture), oil, Alcohol, and thick glass.

   B. Light the laser beam at the marked point on the bowl filled with each material at **45°**.

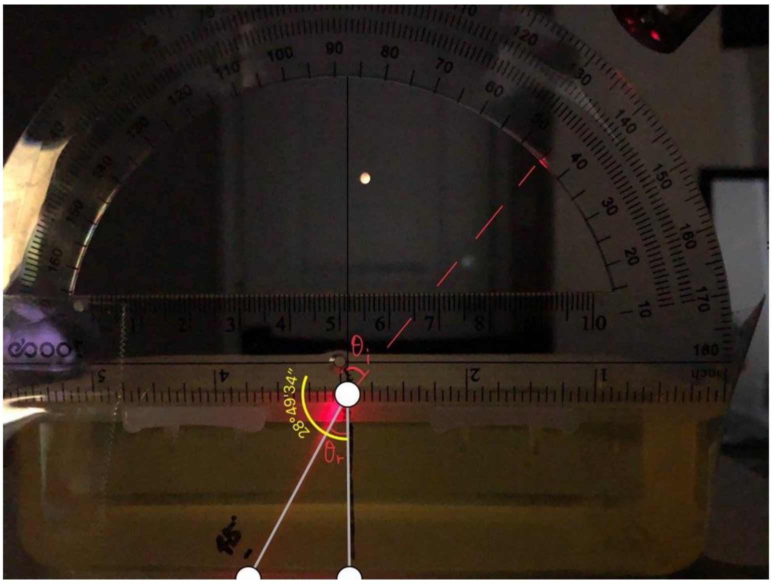
   C. Measure the angle between the refracted laser light and the line of a normal to the surface of the medium.

   D. Calculate the index of refraction of the medium, using Snell's law that is solved for n_2;

The above picture is the setup of the equipment for oil.



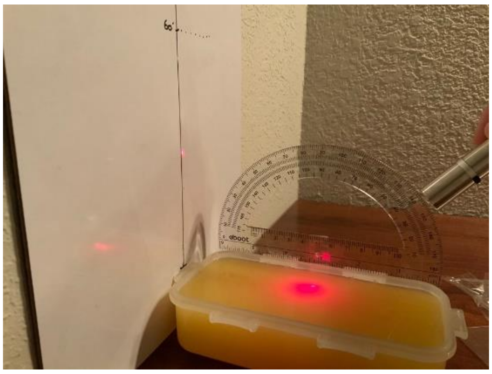
Light in the 45° incident angle to the marked on the surface of oil



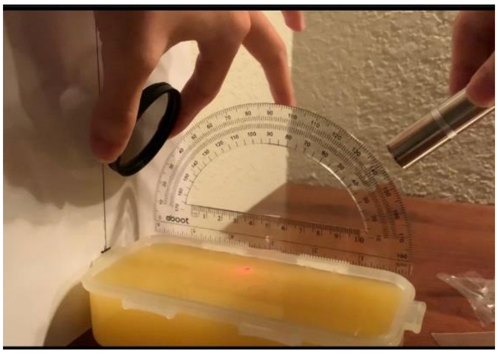
Measure θ_r (angle of the refracted light),

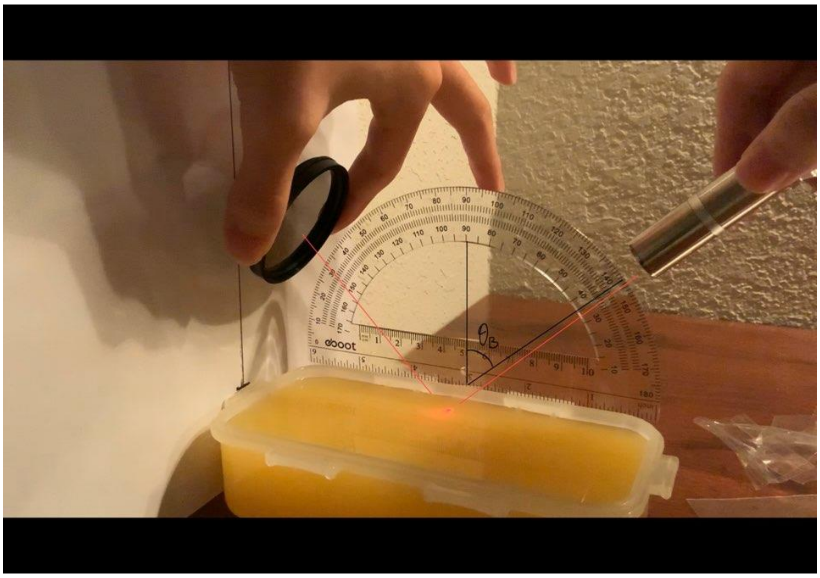
3) Experiment 3: Measure Brewster's angle with various materials.

   A. Set up materials, such as water, juice (mixture), oil, Alcohol, and thick glass.



Observe the reflected light.

 Find what angle of incidence of light produces the weakest intensity after going through the polarizing lens.



Measure the incident angle.

4. Experimental Data and Analysis

1) Experiment 1 – Confirming Snell's Law through Lens Index of Angle of Incidence and Refraction, as well as Critical Angle

Data Table:

Angle of incidence	Value of Sin of Incidence Angle	Angle of Refraction	Value of Sin of Refraction angle (experimental)	Value of sin of Refraction angle (Theoretical)	Percent Difference (%)
15°	0.258	10°	0.173	0.170	1.90
30°	0.499	19°	0.325	0.328	0.919
45°	0.706	28°	0.469	0.464	1.07
60°	0.865	34°	0.558	0.569	1.95
75°	0.965	39°	0.629	0.635	0.949

First, I compare the experimental value of the refracted angle with theoretical value for confirming Snell's law.

To get the value of sin of refraction angle theoretically, I used equation (1), $\frac{sin\theta_2}{sin\theta_1} = \frac{n_1}{n_2}$.

For the value of n_2, I used the accepted value for glass $n_2 = 1.52$, as the second medium I used is made of glass.

For example, if the angle of incidence is 10°, then

$$\sin(10°) \cdot \mathrm{n}_1 = \sin(\theta_r) \cdot n_2$$

$$\Rightarrow 0.258 \cdot 1 = \sin(\theta_r) \cdot 1.52 \qquad (n_1 = 1 \text{ for air})$$

$$\Rightarrow sin(\theta_r) = 0.170.$$

The percent difference between my experimental values and theoretical values were all less than 2%, which shows the consistency of Snell's law.

Critical Angle (Experimental)	**Critical Angle (Theoretical)**	**Percent Difference (%)**
43.0°	41.1°	4.52

Secondly, I compare the experimental critical angle and theoretical critical angle to confirm Snell's law in a different manner.

To get the critical angle theoretically, I use the equation (1) with $\theta_2 = 90$ degrees and solve for θ_1:

$$\theta_1 = \sin^{-1}\left(\frac{n_1}{n_2}\right) \cong \sin^{-1}\left(\frac{1}{n_2}\right), (n_1 = 1 \text{ in air})$$

For n_2=1.52, I find

$$\theta_1 = \sin^{-1}\left(\frac{1}{n_2}\right) = \sin^{-1}\left(\frac{1}{1.52}\right) = 41.1°.$$

The percent difference of 4.52% shows the similarity between the experimental and the theoretical critical angles, confirming the Snell's law.

2) Experiment 2 – Index of Refraction of various materials

Data Table:

Type of Medium	**Angle of Refraction (θ_r) $\pm$ 0.1°**	**Value of Sin of Refraction Angle ($sin\theta_r$)**	**Refractive Index of Medium (n_2)**	**Refractive Index of Medium (accepted value) [3]**

Water	32.4°	0.536	1.3 $\pm$ 0.2	1.33
Orange Juice (mixture)	31.3°	0.520	1.4 $\pm$ 0.2	N/A
Oil	28.5°	0.477	1.5 $\pm$ 0.3	1.47
Alcohol	31.8°	0.528	1.3 $\pm$ 0.2	1.36
Thick glass	29.4°	0.491	1.4 $\pm$ 0.3	1.50

I used the angle of refraction (θ_r) from the previous experiment to calculate the refractive index of medium of each material with the incident angle being 45 degrees.

For example, for oil, $n_2 = \frac{n_1 \sin(\theta_i)}{\sin(\theta_r)} = \frac{1 \sin(45)}{\sin(28.5)} = 1.5$.

Error Analysis

I use equation (3) for the error analysis,

$$n_2 = \frac{n_1 \sin(\theta_i)}{\sin(\theta_r)}$$

The error value is calculated by

$$\Delta n_2 = \sqrt{\left(\frac{\partial n_2}{\partial \theta_i}\right)^2 (\Delta\theta_i)^2 + \left(\frac{\partial n_2}{\partial \theta_r}\right)^2 (\Delta\theta_r)^2} \text{ , where}$$

$$\frac{\partial n_2}{\partial \theta_i} = \frac{n_1 \cos(\theta_i)}{\sin(\theta_r)} \text{ and}$$

$$\frac{\partial n_2}{\partial \theta_r} = \frac{-n_1 \sin(\theta_i)\cos(\theta_r)}{\sin^2(\theta_r)} .$$

In my experiment, pertinent values are

$$n_1 = 1$$

$$\theta_i = 45°$$

$$\Delta\theta_i = 0.05°$$

$$\Delta\theta_r = 0.1°$$

and θ_r is what is in the above table.

3) Experiment 3 – Brewster's Angle

Data Table:

Type of medium (n_2)	Brewster's Angle (θ_B) $\pm$ 0.1°	Theoretical Brewster's Angle (θ_{BT})	Percent Difference (%)
Water	53.6°	53.1°	0.937%
Orange Juice (mixture)	52.8°	53.1° $\pm$ 0.1°	0.567%
Oil	60.2°	55.8°	7.59%
Alcohol	54.1°	53.7°	0.742%
Thick glass	55.7°	56.3°	1.07%

To calculate the theoretical values for Brewster's angle, I used the accepted values of refractive indices [3], and equation (2).

For example, the water has index of refraction: 1.33. So,

$$\theta_{BT} = artan\left(\frac{1.33}{1}\right) = 53.1°$$

Lastly, to get the percent difference between the theoretical Brewster's Angle and the experimental Brewster's Angle, I used $\frac{|\theta_B - \theta_{BT}|}{\frac{\theta_B + \theta_{BT}}{2}}$.

Error Analysis

For orange juice, even the theoretical value of Brewster's angle has some error value, as the value was calculated using the value of the index of refraction that I obtained in the previous experiment. In order to calculate this error, I used equations (2). Since Brewster's angle is given by

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right),$$

the error value is calculated by

$$\Delta\theta = \sqrt{\left(\frac{\partial\theta}{\partial n_2}\right)^2 (\Delta n_2)^2 + \left(\frac{\partial\theta}{\partial n_1}\right)^2 (\Delta n_1)^2} \text{ , where}$$

$$\frac{\partial\theta}{\partial n_2} = \frac{1}{n_1}\sec^2\left(\frac{n_2}{n_1}\right) \text{ and}$$

$$\frac{\partial\theta}{\partial n_1} = -\frac{1}{n_1^2}\sec^2\left(\frac{n_2}{n_1}\right).$$

In my experiment with Orange juice,

$$n_2 = 1.4,$$

$$n_1 = 1, \text{ and}$$

$$\Delta n_2 = 0.2.$$

Therefore,

$$\Delta\theta = 0.1°.$$

5. Conclusion

Overall, the experimental and theoretical values in Brewster's angle experiment showed a very small difference, 1% or less for percent difference, except for oil. In addition, I successfully measured the index of refraction of orange juice, which is not commonly done.

Source of Error

1) In the first experiment, I had marked the angle of incidence on the paper in advance. However, the beam did not necessarily enter the lens exactly at the angle I wanted.

2) If the orientation of the light of the laser beam and the desk are not parallel, the values shown in the angle on the desk does not represent the angle of reflection. There must have been a slight error in this regard.

3) In measuring Brewster's angle, I filled the bowl with the medium to be tested, trying to place the protractor as close as possible to the surface of the medium. However, the starting point of the protractor surface and the surface of the medium would not perfectly match and there would have been some errors.

4) During the experiment, I had to turn the lens several times to see when the light that passed through the lens weakened the most. In the process I may not have found the angle that gave precisely zero intensity after going through the polarizer.

Future Improvement

If I were to continue this experiment, I would like to verify that the angle between the reflected light and the refracted light is 90 degrees, which is a characteristic of Brewster's angle. In my experiment, this angle was hard to observe. In particular, because orange juice is opaque, it was impossible to confirm this angle. Even though other materials are transparent, the surface of the bowl refracted the light again, making it difficult to identify the refracted beam. Therefore, in the future, it would be better to prepare a bowl that is a wider and deeper, and as transparent and thin as possible, in order to check the 90° angle on the side of the bowl.

Another improvement I can make is a better measurement of the intensity of light. As the point of zero intensity was confirmed by naked eyes, it was not as precise as it could have been. To overcome this, I think that the laser light should be much stronger. If the laser light is much stronger, it will help the experiment because the intensity of the light before and after going through the lens it is much more distinguishable.

Reference

[1] "Snell's Law." *Wikipedia*, Wikimedia Foundation, 10 Dec. 2020, en.wikipedia.org/wiki/Snell's_law.

[2] "An Undergraduate Optics Problem – The Brewster Angle." *This Condensed Life*, 20 Apr. 2017, thiscondensedlife.wordpress.com/2017/04/19/an-undergraduate-optics-problem-the-brewster-angle/.

[3] "Brewster's Angle." *Wikipedia*, Wikimedia Foundation, 22 Nov. 2020, en.wikipedia.org/wiki/Brewster's_angle.