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Special topics in Physics: Electricity and Magnetism

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Electricity and Magnetism

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Abstract:

With all the problems humanity faces, every solution seems to require more and more energy. Visionaries and scientists alike seem to agree that one of humanities major hurdles in the future is providing for the enormous need of clean renewable energy. While many mediums to transfer energy exist, electricity is by far the most popular, with petroleum a close second. For electricity, many techniques exist for its generation, but the most common is some type of electromechanical generator. With the increasing need for clean energy, more generating devices will be needed. In order to facilitate this increase it is prudent to understand the electrical-mechanical workings of such a generator in order to better design, build, and repair such devices. This paper will develop some general equations for one possible design for such a device, and use specific values used to calculate a theoretical output. The ultimate goal is to build a working model that matches the design of the formulas derived to see how close a proof of concept matches theory, and to see what differences may result.

1. Introduction.

The electrical part of any electro-mechanical generator, of any type, functions according to the laws of physics described in the third Maxwell equation, also known as Faraday's Law of Induction.

$$\oint_C \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{A}$$

This says that the sum of the E-field over the total distance around a closed loop, otherwise called Voltage, is equal to the negative time rate of change of the total Magnetic flux through the area that the loop contains. While this equation is very descriptive and specific, it does not specify any size, shape, or geometry of these quantities. It only states that the sum of one dot product is equal to the negative time derivative of another dot product. Before this equation can be applied in any useful way, a preliminary geometry must be specified with a relevant description of the different parameters.

The final step of this project is to attempt to produce a working prototype and compare theoretical values to reality. To do this and minimize cost and complexity, readily available parts will be used, and the equations designed around their geometry. Currently, arc shaped magnets are available that have an 8-in OD and 4-in ID and are magnetized through their ¼"thickness. (See fig1.) These magnets have an arc measure of 22.5º each, so 16 magnets grouped together make a complete circular ring. (See fig.2) The magnets are arranged alternating north and south poles around the ring so that when this ring spins it provides an alternating magnetic field. Since electrical induction involves moving a coil of wire through a magnetic field, it is easiest to make the coils stationary and the permanent magnetic field rotate.

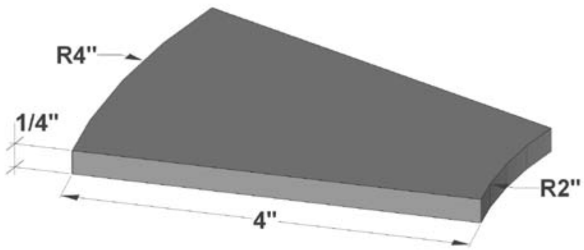

Figure 1

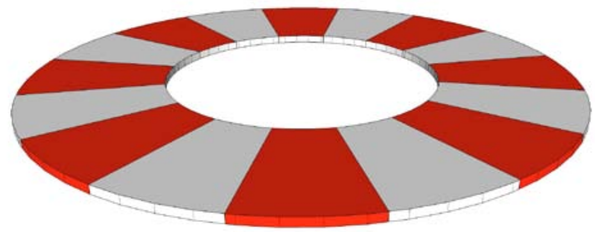

Figure 2

This is beneficial for at least three reasons. First, it avoids the difficulty of requiring electrical brushes to transport the electricity off of a rotating armature. Second, when current flows through the generating coils they will experience a force from the moving magnetic field. Wire coils are designed to be compact and often require structural reinforcement. This is much easier to achieve if they are stationary rather than moving. Third, it is much easier to mechanically balance an object with uniform shape and density like a relatively solid disk, than the irregular shape and density of an armature coil.

This rotating disk of alternating magnets will create the necessary time varying magnetic field which accounts for the right side of Faradays law of induction. However, the left side needs a conducting loop that this time varying magnetic flux will penetrate. To maximize the induced voltage, the magnetic field strength through the coil must also be maximized. Magnetic flux lines exit the magnet normal to its surface, but immediately begin to diverge seeking out the opposite magnetic poles on either side. To get the maximum flux through a loop, that loop must be close to the surface and no larger than the wedge area or it will include oppositely directed flux from the neighboring magnet. Therefore the loop area of the coil should closely match the shape of each magnet arc itself. However, current carrying wires develop hot spots at sharp bends or corners. Extending the loop area slightly inside and outside the arc segment in the radial direction will solve this problem by allowing the wire to bend smoothly.

II. Voltage

To calculate the voltage induced by the moving field we must first find the total magnetic flux through the closed loop as a function of time. The voltage is the negative of the time derivative of this changing flux. To find this flux as a function of time we construct a flux area the size of the coil overlapping 2 of the magnet arc segments. (see Figure 3)

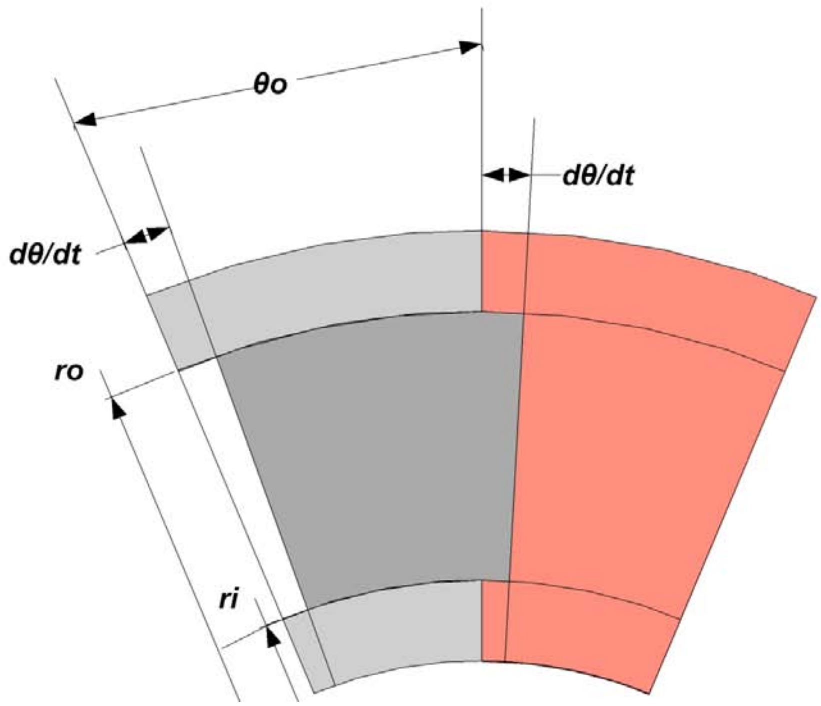

Figure 3

From this graphic we can derive an equation for the flux through the induction coil. Add the total flux contribution from both the light gray and red arc segments that go through the dark gray coil area. Even though the permanent magnet disk is what is actually moving with angular velocity ω, we derive the equations as though the gray coil area is moving to the right. We need the total flux contributions from both sides through the dark gray coil. That is, $\Phi_{total} = \Phi_{left} + \Phi_{right}$. In general, the flux is the number of turns of wire in the loop(n) times the magnetic field(B_o) times the area(A), $\Phi = nB_oA$. With the area expressed in cylindrical coordinates and the nonmoving coil completely centered over the left side, the beginning flux becomes $\Phi_o = nB_o\theta_o(r_o^2 - r_i^2)/2$ with $\theta_o = \pi/8$. Now let the field move with angular velocity ω, field to the left,(or dark gray coil region to the right.) To find the changing flux with time, take time derivative of both sides. Also note that the area over the left arc segment is decreasing so the change in the flux from the left side must be negative. This results in the following derivation…

$$d\Phi_L/dt = -nB_L[(r_o^2 - r_i^2)/2(d\theta/dt)] = -nB_L(r_o^2 - r_i^2)/2\ \omega$$

Separate variables by multiplying by *dt* to get…

$$d\Phi_L = -[n\,\omega\,B_L(r_o^2 - r_i^2)/2]\,dt$$

Integrate left side from Φ_o to $\Phi_{L(t)}$, and right side from 0 to t…

$$\int d\Phi_L = \int -[n\omega B_L(r_o^2 - r_i^2)/2]\,dt \quad = \quad \int d\Phi_L = -[n\omega B_L(r_o^2 - r_i^2)/2]\int dt$$

This gives the total flux from the left arc segment as a function of time…

$\Phi_{L(t)} - \Phi_o = -[n\omega B_L(r_o^2 - r_i^2)/2]t$

$\Phi_{L(t)} \quad = -[n\omega B_L(r_o^2 - r_i^2)/2]t + \Phi_o = -[n\omega B_L((r_o^2 - r_i^2)/2]t + nB_L\theta_o(r_o^2 - r_i^2)/2$

Now a similar derivation for the right side contribution, but this time the flux area over the right arc segment is increasing, so the change in the flux must be positive...

$d\Phi_R/dt = nB_R[(d\theta/dt)(r_o^2 - r_i^2)/2] = n\omega B_R(r_o^2 - r_i^2)/2$

Separate variables by multiplying both sides by *dt* to get…

$d\Phi_R = [n\omega B_R(r_o^2 - r_i^2)/2]dt$

Integrate left side from 0 to $\Phi_{(t)}$, and right side from 0 to t…

$\int d\Phi_R = \int [n\omega B_R\theta_o(r_o^2 - r_i^2)/2]\ dt = \int d\Phi_R = [n\omega B_R\theta_o(r_o^2 - r_i^2)/2] \int dt$

This gives the total flux from the right arc segment as a function of time…

$\Phi_{R(t)} - \Phi_o = [n\omega B_R(r_o^2 - r_i^2)/2]t - 0 = [n\omega B_R(r_o^2 - r_i^2)/2]t$

Adding both flux contributions from each side gives…

$\Phi_{tot}(t) = \Phi_L(t) + \Phi_R(t) = \left(-[n\omega B_L(r_o^2 - r_i^2)/2]t + n\omega B_L\theta_o(r_o^2 - r_i^2)/2\right) + [n\omega B_R(r_o^2 - r_i^2)/2]t$

Substitute $B_L = - B_R$, since the arc segments have same magnitude but opposite field direction…

$\Phi_{tot}(t) = -[n\omega(-B_R)(r_o^2 - r_i^2)/2]t + n\omega(-B_R)\theta_o(r_o^2 - r_i^2)/2 + [n\omega B_R(r_o^2 - r_i^2)/2]t$

Group like terms gives the final equation for $\Phi_{tot}(t)$…

$\Phi_{tot}(t) = 2[n\omega B_R(r_o^2 - r_i^2)/2]t + - nB_L\theta_o(r_o^2 - r_i^2)/2$

It should be noted that this equation is only valid from (t = 0) up to ($t = \theta_o/\omega$) from there the values of B_L and B_R should be swapped and this process repeated and the flux computed again. This will eventually result in a repeating triangular wave with amplitude $nB_L\theta_o(r_o^2 - r_i^2)/2$ and period (Period(T) = $2\ \theta_o/\omega$).

The voltage is then the negative time derivative of this equation. This value alternates as a square wave with magnitude …

$V_{max} = 2[n\omega B_R(r_o^2 - r_i^2)/2]$ (volts)

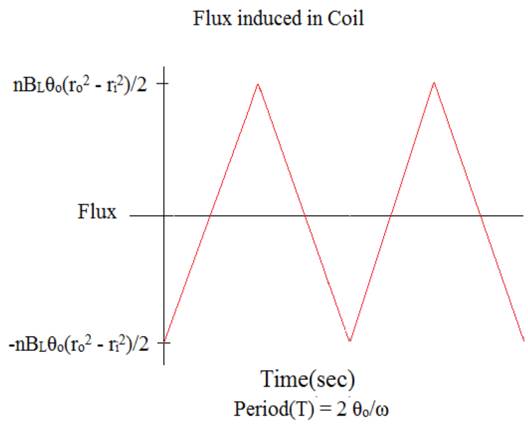

Figure 4.

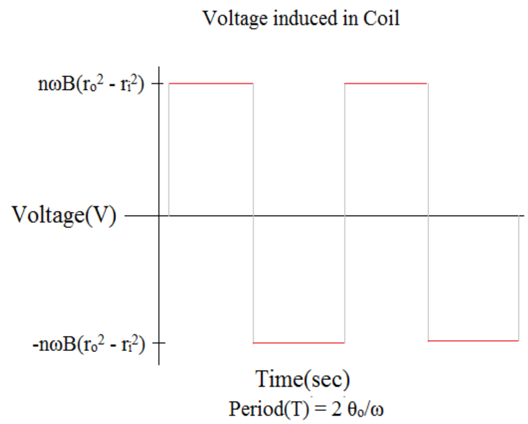

Figure 5.

III. The next challenge is to determine the current in the loop when it is passed through a magnetic field. Thus we need the total resistance of this loop. This follows according to the general equation…

Current = Voltage / Resistance

For a passive device, or a device that is purely resistive, this is a simple analysis. For others this can be more complicated as some have a type of resistance that varies with the frequency of the voltage. This time varying resistance is called impedance and has two forms. The first is called capacitive reactance, and is defined…

$$X_c = 1/\omega C$$

Where C is the capacitance of the device, and ω is frequency in radians per second of the voltage through the device. The second type of reactance is called inductive reactance and is defined…

$$X_L = \omega L$$

Where L is the self inductance of the device, and again ω is the frequency of the voltage through the device. These two reactances along with the resistance combine to give the complete impedance for any device. However for our design here, ω will be in the range of 100 to 500 due to the input rotation of our generator. This makes X_L much greater than Xc, and is therefore the only important component of reactance for our design. To find XL we will need the self inductance of our generating loop of wire. In a classical textbook of physics the formula for the self inductance of a coil of wire is…

$$L = \mu n^2 A l$$

Where n is the number of turns per unit length, A is the cross sectional area, and l is the length of the solenoid coil of wire. This equation is an approximation for the inductance but only for a specific shape. This shape is for the length to be longer than the diameter of the coil. The larger the length to area (L/A) is the better the approximation. When L/A becomes smaller than 1, this approximation breaks down and is no longer useful. For our design, the coil needs to be as small in thickness so it can sit as close to the magnetic field to capture as much magnetic flux as possible. When comparing geometries the induction coil is the opposite shape of that for which the approximation is valid. Instead of a long tube like cylindrical coil of wire, it is short with a large cross sectional area. Methods do exist for calculating inductance of this shape, but the methods are beyond the scope of this discussion. For that reason the results from more involved methods of finding self inductance will referenced, but not derived. As it turns out, the inductance of a generic polygon of wire with perimeter *p* , area *A*, wire radius R may be approximated as…

$$L \approx \mu_o p N^2 \left[\ln\left(2p/R\right) + 0.25 - \ln\left(p^2/A\right) \right] / 2\pi$$

This function is strongly dependent on the perimeter and weakly dependent on the loop area and wire radius. [Grover pp. 60] This can be modified for N turns of wire by multiplying by N^2. Thus for our induction coil the inductance would be as follows…

Perimeter p = $\theta_o r_o + \theta_o r_i + 2(r_o - r_i)$ = $(\pi/16)(4+2) + 2(4-2)$ = 5.178 inches = 0.1296 meters.
Area = $(1/16)[\pi r_o^2 - \pi r_i^2]$ = $(\pi/16)[4^2 - 2^2]$ = 2.356 square inches = 0.001475 square meters.
R(say for 16ga wire) = d/2 = (0.001291)/2 = 0.0006455 meters.

Substitution of these values for a coil of 50 turns gives a self inductance of Henries.

Thus the total current generated is …

$$= V/Z_{tot} = I_{max} = \frac{-[n\omega B_o(r_o^2 - r_i^2)]}{Z_{tot}}$$

Where

$$Z_{tot}^2 = [\omega L - 1/(\omega C)]^2 + R_L^2 = \left[\omega \mu_o p N^2[\ln(2p/R) + 0.25 - \ln(p^2/A)]/2\pi\right]^2 + R_L^2$$

Where $1/(\omega C)$ is zero, and R_L is the internal resistance of the coil. Usually this coil resistance is very small and with very large induction coils a larger number of windings results in a larger resistance, but this is still very small compared to the overall impedance.

IV. To determine the power produced in the induction coil, we must use P = I*V, but this value must be modified to allow for the phase difference. With an inductive load the voltage across the load leads the current by 90 degree phase shift. With a capacitive load the voltage lags the current by 90 degrees. Since we have assumed the capacitance of the induction coil, often called parasitic capacitance, to be negligible we have only inductive reactance. The power is equal to the voltage times the current at the any one instant. Since the voltage and current are not in phase, the maximum voltage and maximum current never occur at the same time, and the power is reduced by a factor of $\text{Cos}(\delta) = Z/R$.

So the maximum power produced is…

$$P_{max} = \frac{V^2}{Z_{tot}} * \text{Cos}(\delta) = \frac{[n\omega B_o(r_o^2 - r_i^2)]^2 * \text{Cos}(\delta)}{Z_{tot}}$$

.

$$P_{max} = \frac{[n\omega B_o(r_o^2 - r_i^2)]^2}{\left(\left[\omega\mu_o p\, N^2[\ln(2p/R) + 0.25 - \ln(p^2/A)]/2\pi\right]^2 + R_L^2\right)^{1/2}}$$

This shows the power output to be directly proportional to ω, and to the strength of the magnetic field B, as one would expect.

V. Another relevant dynamic quantity is the resistive torque. This is important because it determines the loading characteristics of the mechanical input. Most sources of mechanical energy input one might use, such as water or wind, will provide a relatively constant level of input energy. For our purposes that energy will be converted into a relatively constant torque the generator by some mechanical device like a paddle or fan. To utilize this constant torque, the resistive torque of the generator must also be relatively constant for a given angular speed. To ensure this we need to calculate the resistive force of the induction coil and multiply it by the average radius of the resistive force. This will give the resistive torque of the inductive coil.

The resistive force in a generator of this kind comes from the interaction of the magnetic field with the current in the induction coil and obeys the equation…

$$F = q\mathbf{V} \times \mathbf{B} \quad \text{(for our purposes)} = i\mathbf{l} \times \mathbf{B}$$

Where i is the current in the wire, n is the number of loops, l is the length of the wire segment and B is the strength of the magnetic field. Since the force is constant along the radial direction the average radius can simply be taken as the average distance of the two radii, in this case, 3 inches. So torque is…

$$\text{Torque} = r_{avg}\,(i\mathbf{l} \times \mathbf{B}) = r_{avg}\mathbf{l}\frac{[n\omega B_o(r_o^2 - r_i^2)]}{Z_{tot}} \times \mathbf{B}$$

The nature of the vector cross product is such that the resistive force is always in the direction opposite to the rotational motion of the disk. The above Torque is only for one side of the induction coil, so the actual resistive force for one coil is twice this value.

$$\text{Torque max} = 2 * r_{avg}\,\mathbf{l}\,\frac{[n\omega B_o(r_o^2 - r_i^2)]}{Z_{tot}} \times \mathbf{B}$$

Since the voltage is a square wave, the current is also, but it lags the voltage by nearly 90 degrees. The power output is also a square wave with a nonzero value where the voltage and current waveforms overlap, but it has a pulse width half of the voltage pulse width while still the same period. This pattern produces an intermittent power output. To solve this problem, additional induction coils will be placed in the proper locations around the structure so that the power output from these coils does not coincide with any others. This will create a continuous power generation from the generator with different parts coming from different coils.

VI. Mutual inductance

As described above, more than one induction coil is actually used in a generator so that the majority of the moving flux can be harnessed for power generation. However, when these coils get closer to one another, a phenomenon known as mutual inductance becomes a factor. This occurs when the changing magnetic field through one coil area penetrates the loop area of a neighboring coil and induces a voltage. The geometrical arrangement of these induction coils must be such that any mutual inductive effect from one coil to another is constructive and creates an adding effect to the total flux in a loop. Otherwise the voltages induced in the coils will oppose one another reducing the total potential output. The complexity of this is beyond that of this discussion, but suffice it to say that this must be taken into account when designing any type of electromechanical generator.

Conclusion

Through the development of these equations from the geometry proposed I have a new respect for those involved with the design of these devices. The number of factors that must be considered are enormous and the derivations are complex. Everything is more complicated rather than less. In class, we deal with the theoretical ideals and take advantage of its simplicity, but in practice these ideal assumptions cannot be dismissed. This discussion is really a discovery of the complexity of the process of designing a generator, and really serves as a stepping stone to a more in depth attempt where more precise derivation is done.